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Title: Some Contributions to Fuzzy Ideals in Algebraic System

Abstract

Abstract algebra is the study of algebraic structures. Such a structure consists of a set together with one or more binary operations, which are required to satisfy certain axioms. In modern era of mathematics, the topic "Fuzzy ideals in different Algebraic Structures" has become a very rich area of research for Algebraists. Since fuzzy set was introduced, several extensions have been developed, hesitant fuzzy set is a recent extension of fuzzy sets. Hesitant fuzzy sets have attracted the attention of many researchers in a short period of time because hesitant situations are very common in different real world problems. The present thesis is divided into seven chapters:

Chapter 1. The first chapter is introductory in nature. In this chapter, some basic definitions and results have been stated which we shall use in the subsequent chapters of the thesis. Most of these results and definitions are available in standard references on the subject. We have collected them for ready references and also fix up our notations and terminology. This also serves the purpose of making the thesis as much self-contained as possible.

Chapter 2. In this chapter, we apply fuzzy sets in hyperstructure theory. We study the concept of fuzzy hyperideals, fuzzy prime hyperideals and fuzzy semiprime hyperideals in ordered ternary semihypergroups and explore some related properties. We introduce the concept of ordered fuzzy point in ordered ternary semihypergroups. We also give a number of characterizations for regular and an intra-regular ordered ternary semihypergroups by using the concept of fuzzy hyperideals. Moreover, in support of our discussion, we discuss some examples also.

Chapter 3. In this chapter, we study some properties of hesitant fuzzy ideals and hesitant fuzzy bi-ideals in a semigroup and discuss their characterizations. Also, we introduce hesitant fuzzy interior ideal in a semigroup. It is proved that in a semigroup a hesitant fuzzy ideal is a hesitant fuzzy interior ideal but the converse is not true. Moreover, we prove that in regular and in intra-regular semigroups the hesitant fuzzy ideals and the hesitant fuzzy interior ideals coincide. Finally, we introduce hesitant fuzzy ideals on ternary semigroups and define hesitant fuzzy left (resp., lateral, right)ideals on ternary semigroups. Moreover, in support of our discussion, we discuss some examples also. The contents of this chapter are accepted for publication in International Journal of Applied Mathematics.

Chapter 4. In this chapter, we introduce the hesitant fuzzy ideal, hesitant fuzzy biideal and hesitant fuzzy interior ideal in ordered semigroup. We also prove a number of results based on (left, right, two-sided, interior and bi-) ideals of ordered semigroup. We also study hesitant fuzzy semiprime ideals, hesitant fuzzy left simple, hesitant fuzzy right simple and hesitant fuzzy simple in ordered semigroups. Then we characterize weakly regular ordered semigroups by means of idempotent hesitant fuzzy ideals. We also characterize semi-simple ordered semigroups in terms of hesitant fuzzy ideals. Finally, a relation between hesitant fuzzy (left, right and bi-) ideal in terms of intra-regular and left-weakly regular ordered semigroup is studied. The main results of this chapter are published in Proceedings of IIRAJ International conference (ICCE-SEM-2017) GIFT, Bhubaneswar, Khurda, India.

Chapter 5. In this chapter, we introduce the notions of extension of hesitant fuzzy ideals, hesitant fuzzy prime ideals, hesitant fuzzy semiprime ideals and hesitant 3-prime fuzzy ideals in ordered semigroups. We discuss the relationship between fuzzy prime ideal and 3-prime ideals in ordered semigroups by means of the extensions hesitant fuzzy ideals. We give an example to show that 3-prime hesitant fuzzy ideal is not necessarily prime in an ordered semigroup. Moreover, we give examples in each section in support of our discussion. The contents of this chapter are accepted for publication in Turkic world mathematical society journal.

Chapter 6. In this chapter, we apply hesitant fuzzy set theory in Γ -semigroups and ordered Γ -semigroups. The hesitant fuzzy ideals in Γ -semigroups as well as ordered Γ -semigroups are introduced and some related properties are explored. Using this concept, some characterizations on hesitant fuzzy left (right and bi-) ideals are given. The hesitant fuzzy interior ideal in Γ -semigroups and ordered Γ -semigroups are defined and their related properties are studied. We also prove a number of subsidiary results and most notably we prove that, a non-empty hesitant fuzzy subset H of a Γ -semigroup S is hesitant fuzzy ideal of S if and only if the T -cut \mathcal{H}_T of \mathcal{H} is an ideal of S for every $T \subseteq P([0, 1])$, provided \mathcal{H}_T is non-empty. Moreover, in support of our discussion, we discuss some examples also. The contents of this chapter are accepted for publication in Italian Journal of Pure and Applied Mathematics.

Chapter 7. In this chapter, we have collected the findings of thesis and write the conclusion for further work.

In the end we have given a bibliography arranged alphabetically which by no means is exhaustive on the subject. In fact only those works have been listed which have been referred in the thesis by a serial number in a square bracket.