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ABSTRACT

In the present thesis entitled **``A Study On Power Series Rings And Modules"**, we have studied skew polynomial, skew power series, skew Laurent power series, skew generalized power series over principally quasi-Baer rings and modules. The study of Baer rings (i.e. rings in which the right annihilator of every nonempty subset is generated by an idempotent) was initiated by Kaplansky. As a generalization of Baer rings, Clark defined quasi-Baer rings (i.e. rings in which the right annihilator of every ideal is generated by an idempotent). In 1974, Armendariz considered the behaviour of a polynomial ring over a Baer ring. Further, many authors investigated Baerness and quasi-Baerness of a ring for different rings and their extensions including skew polynomials, formal power series, skew power series, skew Laurent polynomial etc. Birkenmeier et al. gave a new direction to the development of Baer ring and introduced the concept of principally quasi-Baer ring (or simply, p.q. Baer ring). A ring $\mathcal{R}$ is called right principally quasi-Baer if the right annihilator of a principal right ideal of $\mathcal{R}$ is generated by an idempotent. Afterwards, many results have been proved for skew polynomial and skew power series over p.q. Baer rings. Here, we investigate power series and its extensions over p.q. Baer rings and modules and also generalize some of the important results. Further we analyze the concept of α qusi Baer and α principally quasi Baer ring and give some results for skew power series ring over α -quasi Baer ring. Also, we study the concept of skew generalized power series and analyze the relationship between skew generalized power series ring and some abstract ring structures.

Following are the important results of our thesis.

- Let $\mathcal{R}$ be an α -weakly rigid ring with $S_l(\mathcal{R}) \subseteq C(\mathcal{R})$ then $\mathcal{R}[[x,\alpha]]$ is right p.q. Baer ring if and only if $\mathcal{R}$ is right p.q. Baer ring and any countable family of idempotents in $\mathcal{R}$ has generalized join in $Id(\mathcal{R})$.

- Let $\mathcal{R}$ be a ring with IFP Property, $\alpha : \mathcal{R} \to \mathcal{R}$ be an endomorphism of $\mathcal{R}$ and let $\mathcal{R}_{\mathcal{R}}$ be an α -compatible module then $\mathcal{R}[[x,\alpha]]$ is right p.q. Baer if and only if $\mathcal{R}$ is right p.q. Baer and any countable subset of $S_r(\mathcal{R})$ has generalized countable join.

- Let $\mathcal{R}$ be an α -SPS Armendariz ring. Then the following conditions are equivalent: (i) $\mathcal{R}[[x;\alpha]]$ is right $p.p.$ -ring; (ii) $\mathcal{R}$ is right $p.p.$ ring and any countable

family of idempotents in $\mathcal{R}$ has a join in $Id(\mathcal{R})$.

- Let $\mathcal{M}_{\mathcal{R}}$ be an α -weakly rigid module and α be an automorphism of $\mathcal{R}$ then the following conditions hold: (i) If $\mathcal{M}[x,x^{-1};\alpha]_{\mathcal{R}[x,x^{-1};\alpha]}$ is a p.q. Baer then $\mathcal{M}_{\mathcal{R}}$ is p.q. Baer, converse is true if in addition $\mathcal{M}_{\mathcal{R}}$ is α -reduced; (ii) If $\mathcal{M}[[x,x^{-1};\alpha]]_{\mathcal{R}[[x,x^{-1};\alpha]]}$ is p.q. Baer then $\mathcal{M}_{\mathcal{R}}$ is p.q. Baer.

- Let $\mathcal{M}_{\mathcal{R}}$ be an α -weakly rigid module. If the following conditions hold: (i) $r_{\mathcal{R}[[x;\alpha]]}(m(x)\mathcal{R}[[x;\alpha]])$ is left s -unital, for any $m(x) \in \mathcal{M}[[x;\alpha]]_{\mathcal{R}[[x;\alpha]]}$, (ii) $r_{\mathcal{R}}(m\mathcal{R})$ is left s -unital, for any $m \in \mathcal{R}$, (iii) $\mathcal{M}_{\mathcal{R}}$ is α -skew quasi-Armendariz of power series type or skew quasi-Armendariz of power series type, then $(i) \Rightarrow (ii) \Rightarrow (iii)$.

- Let $\mathcal{R}$ be an α -weakly rigid ring. If $\mathcal{R}$ is α -quasi Baer ring then $\mathcal{R}[[x;\alpha]]$ is an α -quasi-Baer ring.

- Let $\mathcal{R}$ be a quasi α -Armendariz ring. If $\mathcal{R}$ is α -quasi Baer ring then $\mathcal{R}[[x;\alpha]]$ is a quasi-Baer ring.

- Let $\mathcal{R}$ be a $(\mathcal{S},\omega)$ - Armendariz ring, $(\mathcal{S},\leq)$ be a strictly ordered monoid satisfying the condition $1 \leq s$ for every $s \in \mathcal{S}$ and $\omega : \mathcal{S} \to End(\mathcal{R})$ a monoid homomorphism. Assume that $\mathcal{R}$ is $\mathcal{S}$ -compatible, then $\mathcal{R}[[\mathcal{S},\omega]]$ is a weakly $p.p.$ -ring if and only if $\mathcal{R}$ is a weakly $p.p.$ -ring.

- Let $\mathcal{R}$ be a ring without non-zero divisors, $(\mathcal{S},\leq)$ a strictly ordered monoid and $\omega : S \to End(\mathcal{R})$ a monoid homomorphism. Assume that order $\leq$ can be refined to strict total order $\preceq$ on $\mathcal{S}$. Then $\mathcal{R}[[\mathcal{S},\omega]]$ is Dedekind finite.

- Let $\mathcal{R}$ be a ring, $(\mathcal{S},\leq)$ a strictly ordered monoid satisfying the condition $1 \leq s$ for every $s \in \mathcal{S}$ and $\omega : \mathcal{S} \to End(\mathcal{R})$ a monoid homomorphism. Then $\mathcal{R}[[\mathcal{S},\omega]]$ is a clean ring if and only if $\mathcal{R}$ is a clean ring.