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Title of the Thesis:- Geometry of lightlike submanifolds in semi-Riemannian manifolds

Abstract

In the process of generalization of submanifold theory from Riemannian manifolds to semi-Riemannian manifolds, degenerate(lightlike) submanifolds arise naturally in the semi-Riemannian category. The study of these submanifolds is interesting and complicated due to the fact that the intersection of normal bundle and tangent bundle is non-empty. This unique feature makes the study of lightlike submanifolds different from the study of non-degenerate submanifolds. Degenerate submanifolds are playing an increasingly important role in quantum theory and string theory, as the action and field equations of particles and strings often do not depend on the inverse metric and are well defined even when the metric becomes degenerate[1]. For example, an extension of Einstein's gravitational theory which contains degenerate metrics as possible solutions might lead to space-times with no causal structure[2]. The growing importance of lightlike submanifolds in other disciplines, in particular their extensive use in general relativity and very limited information available on these submanifolds, is the motivation behind selecting the topic for the present Thesis.

The present thesis comprises five chapters. The first chapter is introductory, where we have summarized some definitions and results from theory of semi-Riemannian manifolds, lightlike submanifolds and semi Riemannnian product manifolds which are of relevance to the subsequent chapters. It also includes the basic information on warped product submanifolds. The last section of this chapter deals with the contact screen Cauchy Riemannian (SCR)lightlike submanifolds required to formulate the definition of SCR-lightlike warped submanifolds in an indefinite Sasakian manifold.

Chapter II, begins by defining screen Cauchy-Riemann(SCR) lightlike submanifolds in a semi-Riemannian product manifold supported by examples. We obtain integrability conditions for the distributions and investigate the geometry of leaves of distributions involved. In section 2.3, we establish that there exists no totally umbilical and curvature invariant proper SCR-lightlike submanifold in any semi-Riemannian product real space form. The necessary and sufficient conditions for SCR-lightlike submanifolds to be locally lightlike Riemannian product manifold are obtained in section 2.4. The contents of this chapter are published in **Int. Elect. J. Geom., 4(2011)2, 141-154.**

In chapter III, we study totally umbilical screen transversal lightlike submanifolds immersed in a semi-Riemannian product manifold and obtain necessary and sufficient conditions for induced connection ∇ on a totally umbilical radical screen transversal

lightlike submanifold to be metric connection. We prove a theorem which classifies totally umbilical ST-anti-invariant lightlike submanifold immersed in a semi-Riemannian product manifold. The contents of this chapter are published in **Advances in Pure Mathematics.,2(2012),285-290.**

In chapter IV, the concept of hemi-slant lightlike submanifolds immersed in indefinite Kaehler manifolds is introduced. In section 4.2, the basic properties of such submanifolds are investigated and the definition is supported by an example. We also establish two characterization theorems for the existence of hemi-slant lightlike submanifolds. We obtain integrability conditions for the distributions involved in the definition of hemi-slant lightlike submanifolds and investigate the geometry of leaves of these distributions in section 4.3. We also obtain a geometric condition under which induced connection ∇ on M is a metric connection. The necessary and sufficient conditions for hemi-slant lightlike submanifolds to be a hemi-slant lightlike product are discussed in section 4.4. The contents of this chapter are published in **Journal of Advanced Scientific Research in Dynamical and Control Systems,4(2012)(3),10-22.**

In chapter V, we study SCR-lightlike warped submanifolds in an indefinite Sasakian manifold. In section 5.2, we prove that every warped product SCR-lightlike submanifold $N_\perp \times_\lambda N_T$ in an indefinite Sasakian manifold is a SCR-lightlike product and give an example in support of the existence of SCR-lightlike warped product of the type $N_T \times_\lambda N_\perp$. In section 5.3, we obtained some characterizations of such submanifolds in indefinite Sasakian manifold. The contents of this chapter are accepted for publication in **Journal of Tensor society, India**

References

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