Name of Scholar - **Roshan Lal**
Name of Supervisor - **Prof. (Mrs.) K.K.Dewan**
Name of Co-Supervisor- **None**
Department - **Mathematics**
Title of Thesis - **Markov's and Bernstein type of inequalities and related problems**

ABSTRACT

The thesis entitled, "**MARKOV'S AND BERNSTEIN TYPE OF INEQUALITIES AND RELATED PROBLEMS**" consists of four chapters. In the first chapter, we study the problems concerning maximum modulus of polynomials. In the second chapter we deal with inequalities for polynomials and their ordinary derivatives. In the third chapter we investigate certain problems concerning the inequalities for the polar derivatives of a polynomial. The fourth chapter deals with the location of the zeros of the real and complex polynomials as well as analytic functions.

If $p(z)=\sum_{\nu=0}^{n} a_\nu z^\nu$ is a polynomial of degree n and

$$M(p,R)=\max_{|z|=R}|p(z)| \text{ , then}$$

$$M(p',1)\le nM(p,1) \tag{1}$$

$$M(p,R)\le R^n M(p,1) \qquad R>1 \tag{2}$$

$$M(p,r)\ge r^n M(p,1) \qquad r<1. \tag{3}$$

Inequality (1) is a well known result known as Bernstein's inequality, whereas inequality (2) is a simple deduction from the Maximum Modulus Principle. Inequality (3) is due to Zarantonello and Varga.

In Chapter 1, firstly, we prove the best possible results for the polynomials having no zeros in $|z|<k, k\ge 1,\ 1\le R\le k^2$, and $R\ge k^2$, and these improve upon the results of Govil, Qazi and Rahman. Further, we prove a result which is again best possible for the polynomials not vanishing in $|z|<k, k\ge 1$ and $0\le \rho<1$. This improves upon a result due to Govil, Qazi and Rahman. The next result is for the class of polynomials not vanishing in $|z|<k,\ 0<k\le 1$ and $0\le \rho\le k^2$, is a complement to the previous result. The next two results are for the class of polynomials not vanishing $|z|<k,\ 0<k<1$ and $k^2\le \rho\le k,$ are supplement to each other and also improve upon the results due to Govil, Qazi and Rahman. We also generalize a result due to Mir for the class of lacunary type of polynomials. Besides these results, some other related results have also been proved.

In Chapter 2, firstly, we prove a result for the class of polynomials having all zeros in $|z|\le k, k\le 1,$ which for a particular case improves upon a theorem due to Aziz and Zargar. Next, we prove an interesting result which improves upon a result proved by Dewan and Mir. The next result generalizes the previous result for lacunary type of polynomials and also improves upon a result due to Dewan, Singh and Lal. The next result for the class of polynomials having their s-fold zeros at the origin gives an improvement of a result due to Mir and also reduces to the results of Govil and Aziz and Dawood. We next prove a result for

the class of polynomials having s-fold zeros at the origin and rest of the zeros on $|z| = k,\ k \le 1$. Further, by involving the coefficients of the polynomials, we prove a result that improves upon a result due to Dewan and Bidkham. Finally, we prove a result that is complement to previous result.

In Chapter 3, firstly, we prove a result for the polar derivative of lacunary type of polynomials having all its zeros in $|z| \le k,\ k \le 1,$ with s-fold zeros at the origin. This result refines upon a result due to Mir and also a result due to Aziz and Shah. By introducing coefficients of the polynomials, we next prove a result that extends a result due to Govil to polar derivative. If we involve the coefficients and apply the previous result to $q(z) \equiv z^n \overline{p\left(\frac{1}{\bar{z}}\right)},$ then we have proved a result on polar derivative for the polynomials not vanishing in $|z| < k,\ k \ge 1.$ Our next result is for the polar derivative of the polynomials not vanishing in $|z| < k,\ k \le 1.$ Next, we improve upon a result due to Aziz for the class of polynomials not vanishing in $|z| < k,\ k \ge 1.$ Some other related results have also been proved.

The Chapter 4 is on location of zeros of polynomials. Here we firstly, prove a result for real polynomial, which gives an improvement of a result due to Joyal, Labelle and Rahman. The next result gives us the bound for maximum number of zeros that can lie in a ring-shaped region, for complex polynomial when its real and imaginary parts are taken into account separately. Our next result generalizes the first result of this chapter for polynomials with complex coefficients which improve upon a result due to Govil and Rahman. We have proved two results for the analytic functions which give a zero free region for them which also generalize a result of Shah and Liman. Next we prove a result that generalizes as well as improves upon a result due to Jain and also a result due to Joyal, Labelle and Rahman and Gardner and Govil under certain restrictions.

Findings- In our research work we have obtained several best possible results, generalizing and improving upon many earlier well known results. Results for maximum modulus of polynomials and its derivative (in terms of its degree and maximum modulus on unit circle) not vanishing in a disc of radius greater than unity as well as less than unity are obtained. Some results for polar derivative of polynomial are also obtained. Some interesting results for the location of zeros of polynomial and analytic function in terms of coefficients, under certain restrictions are obtained improving upon some earlier well known results. Discs containing all the zeros, no zeros of polynomial are obtained. Ring shaped regions containing maximum and minimum number of zeros of polynomial are also obtained.