

## Details

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**Name of the Ph.D. Scholar: Naveed Ahmad Shah**

**Name of the Ph.D. Supervisor: Prof. M.A.H. Ahsan**

**Name of department: Physics**

**Title of the thesis: Modified Dirac Structure in Graphene**

## Abstract

The low energy charge carriers in graphene, which are the electron quasiparticles, are described by a Dirac-like equation, where the Fermi velocity,  $v_F$ , replaces the velocity of light,  $c$ . This analogous realization of Dirac structure allows for the simulation of quantum electrodynamics (QED), and makes graphene a test-bed for relativistic theories.

In the thesis, we explore the possibility of using graphene as an analogue of high energy physics beyond QED, particularly for analogue condensed matter simulation of modifications of the Dirac equation, which come from different theories in high energy physics.

One such modification is predicted by specific minimal length scale quantum gravity models. The minimal length scale is a common feature of black hole physics and most of the candidate theories for quantum gravity such as string theory and loop quantum gravity. Quantum mechanically the minimal length is understood as a nonzero minimal uncertainty in position measurements, and the corresponding modification of Heisenberg's uncertainty principle to a generalized uncertainty principle (GUP), which is encoded in modifications of the commutation relations. The fact that the quasiparticles in graphene obey the Dirac-like equation and the underlying structure of the graphene lattice is discrete, provides a hint that the tight binding model for graphene might realize an analogous GUP-modified Dirac structure.

We show that the tight binding Hamiltonian, taking into consideration both the nearest-neighbor ( $nn$ ) and next-to-nearest neighbor ( $nnn$ ) atomic hopping, describing the dynamics of the charge carriers in particular angular regions of the graphene Brillouin zone, is precisely of the form of the GUP-modified Dirac Hamiltonian. Along, with the Fermi velocity,  $v_F$ , replacing the velocity of light, the lattice length scale plays an analogous role of the minimal length scale. In the context of graphene, the dispersion relation becomes asymmetric with respect to the energy of the conduction and the valence bands on including the  $nnn$  atomic hopping. This corresponds to the Lorentz symmetry breaking of the modified Dirac structure through the modifications of the commutator relations and the resulting GUP.

Using this correspondence we then analyze the effect of the  $nnn$  atomic hopping on the theoretically well studied and experimentally demonstrated phenomena of Klein tunneling in graphene. Our prediction agrees with the experiments where a perfect transmission of the quasiparticles incident normally on a potential barrier is observed. We predict that a perfect transmission will also be observed at the angles around which the generalized Dirac structure is realized. These angles differ from the angles, predicted theoretically, when the  $nnn$  hopping contributions are not taken into account. This effect, we argue, thus simulates the effect of the discrete topology of space on the Klein tunneling, and propose an experiment wherein the effect can be detected.

We construct a further generalization of the Dirac structure by considering the tight binding Hamiltonian valid for all the angular regions of the Brillouin zone. We observe that this structure not only simulates the Lorentz symmetry breaking but also the rotational symmetry

breaking and the associated modification of the Dirac equation as expected to occur in many quantum gravity theories such as quantum gravity and causal dynamical triangulation. Moreover, this structure is more accurate and necessary for studying phenomena in graphene at higher energies. We thus explicitly calculate the dispersion relation and obtain the solutions i.e. the wavefunctions from the Hamiltonian.

We thus show that the corresponding emergent field theory in graphene is a table-top realization of the minimal length quantum gravity scenarios. Thus the remarkable result is that in going beyond the low-energy limit in graphene, not only does the correspondence with QED phenomena survive, but rather it is a way to obtain experimental signatures of quantum gravity corrections to QED phenomena, although in their analogue realizations. From the point of view of graphene, more importantly, the generalized Dirac structure provides analytic methods to analyze the high energy corrections to specific QED-like phenomena in graphene.

We also use graphene to simulate the Lorentz invariance violating modifications to Dirac equation, predicted by Very Special Relativity (VSR). We observe that the otherwise massless quasiparticles in graphene, like their high counterparts – massless Dirac fermions, acquire a tiny yet finite mass in the VSR formalism. Further having shown the emergence of the non-Abelian gauge fields in graphene from the tight binding model in which the hopping energies are modulated by time independent elastic strain, and constructing the theory in a Lorentz covariant fashion, we study the theory in the VSR context. We show that the gauge field induced by the time independent elastic strain acquires mass. This is again reminiscent of the VSR effect that has been shown to be true for non-Abelian gauge theories.

Further, we study the VSR modification of the non-Abelian gauge theory realized in graphene. These non-Abelian gauge fields result from the time dependent elastic strain. In this case we use the quantum field theory formalism for curved spacetimes to model the strain deformed graphene by a metric with curvature in the spatial dimensions. Here, as in the case with time independent strain, the non-Abelian gauge fields are rendered massive in the VSR-covariant formalism. The induced mass is determined by the strength of VSR modification. We observe that the theory describing the fermionic fields coupled to the non-abelian gauge fields enjoys gauge symmetry. We quantize the theory using the BRST procedure by fixing these gauge symmetries by adding gauge fixing and ghost terms to the associated Lagrangian.

**Keywords:** Graphene, Klein tunneling, Generalized Uncertainty principle, Analogue quantum gravity, Very special relativity, Gauge fields, BRST.